# Theory of Reflectionless Scattering Modes


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**Abstract:**
We present a theory of reflectionless scattering modes, a novel kind of resonance phenomenon which describes unidirectional and bidirectional, reflectionless transmission resonances, and also reflectionless mode conversion for arbitrary photonic structures in any dimension.


We develop the theory of a special type of scattering state in which a certain set of asymptotic channels are chosen as the desired input channels and the complementary set of channels as the output channels and there is zero reflection back into the input channels [1]. We show that in general an infinite number of such solutions exist at discrete frequencies in the complex ω or energy plane for any choice of the input/output sets. Our results apply to linear electromagnetic and acoustic wave scattering and also to quantum scattering. We refer to such states as reflection-zeros (R-zeros) when they occur off the real axis and as Reflectionless Scattering Modes (RSMs), when they are tuned to the real axis, and exist as steady-state solutions. Such reflectionless behavior requires a specific monochromatic input wavefront, corresponding to an eigenvector of a generalized reflection matrix with eigenvalue zero. RSMs may be tuned to the real axis either by tuning parameters of the scatterer that do not break flux-conservation (index tuning) or by adding loss or gain (non-hermitian tuning). We show that in general only a single continuous parameter needs to be tuned to create an RSM for a given choice of input/output channels, *and that such RSMs exist in all dimensions and even for finite scatterers in free space.* A coupled mode analysis shows that RSMs are the result of a generalized type of critical coupling, valid in all dimensions and arbitrary geometries.

A symmetry analysis of R-zeros and RSMs implies that RSMs of hermitian cavities are bidirectional in the sense that input and output channels can be interchanged and the resulting state will also be an RSM at the same frequency. RSMs of non-hermitian cavities are generically unidirectional and do not satisfy this interchange symmetry. Non-hermitian systems with PT -symmetry are required to have unidirectional R-zeros in complex conjugate pairs, implying that for small T -breaking their R-zeros occur at real frequency, without the need of any parameter tuning. This explains the widely observed existence of unidirectional unit transmission resonances in one-dimensional PT systems and their disappearance at high frequency, as a result of spontaneous PT-symmetry breaking. A new type of exceptional point is shown to occur at this PT transition, leading to a visible change in the lineshape of the reflection zero. Surprisingly, we also identify a similar but distinct exceptional point associated with a spontaneous symmetry breaking transition in systems with P and T symmetry, but no gain or loss.

Our results apply to any type of scattering structure, with or without P or T symmetry, and with any number and type of asymptotic scattering channels (e.g. free space or waveguides). We outline and implement a general technique for solving such problems, which shows promise for designing useful photonic devices.

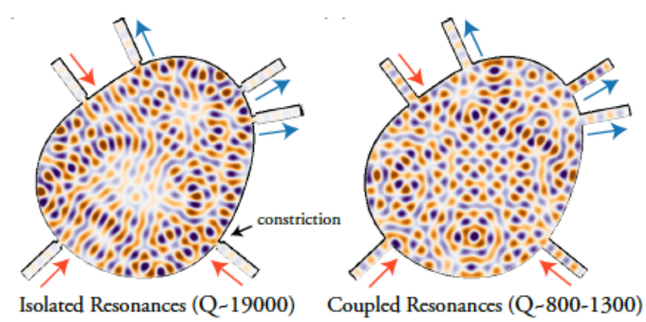


Fig. 1. Reflectionless Scattering Modes for a six-port waveguide junction with three input waveguides and three output waveguides for the case of weak coupling (left ), and strong coupling (right) to the cavity/junction.


[1] Theory of Reflectionless Scattering Modes, W. R. Sweeney, C.-W. Hsu, and A. D. Stone, arxiv arXiv:1909.04017 ([2019].