Metamaterials on a carousel – a journey in a non-inertial landscape

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Moving nano-strauctures and metamaterials may serve as a platform for new non-reciprocal devices such as circulators and isolators [1-5], as well as for a new family of motion-sensors. Of particular interest is the case of structures rotating at a given angular velocity Ω, as rotation is inherently non-inertial thus it exhibits a wider class of unique physical phenomena. Furthermore, compare to linear translation, rotation can be manifested in smaller physical settings. Finally, we point out that the study of the electrodynamics of rotating structures as observed in their *rest frame of reference*, say $\mathbb{R}^{\Omega}$, may be of great interest for several reasons. First, this frame of reference alleviates the need to contend with the computational complexity associated with moving boundaries. The transformation of the incident/scattered field from the inertial/Lab frame $\mathbb{R}^{0}$ to $\mathbb{R}^{\Omega}$, and back, can be done using standard, explicit transformations [6], while the significantly more difficult task of solving the scattering problem itself – the heart of the matter – incorporates non-moving boundaries in $\mathbb{R}^{\Omega}$. Second, this frame of reference is the natural one for the analysis of rotation sensors/optical gyroscopes for navigation, since the rotation estimates must be obtained at the rotating platform itself.

For slow rotation, Maxwell's equations in $\mathbb{R}^{\Omega}$ take on the familiar form that holds in $\mathbb{R}^{0}$, where the effect of rotation is manifested only via the constitutive relations that takes on the form of an elaborated Tellegen's medium [7], that depends on location as well as on the rotation rate. Crucial for the analysis of rotating metamaterials, is the development of polarizability theory and discrete-dipole approximations (DDA) that hold in a structure whose background medium, as well as its constituents, satisfy the aforementioned constitutive relations. It has been shown in [8] that for two-dimensional problems Maxwell's equations can be rigorously separated into independent TM and TE fields, despite the apparent complexity induced by rotation, and a rigorous Green's function formulation in $\mathbb{R}^{\Omega}$ has been developed for both polarizations. In this work, we use this formulation to develop polarizability theory and DDA in $\mathbb{R}^{\Omega}$. The effect of rotation on the polarizability of dielectric particles, as observed in $\mathbb{R}^{\Omega}$, will be studied and discussed. Emphasis will be made to express the polarizability in $\mathbb{R}^{\Omega}$, $\alpha(\Omega)$, in terms of the conventional polarizability in $\mathbb{R}^{0}$, $\alpha(0)$ (i.e. the polarizability of non-rotating particle, observed in the lab frame), plus simplified analytical correction terms for the rotation footprint. We then use the newly developed polarizability theory and DDA, in conjunction of the Green's function in $\mathbb{R}^{\Omega}$, to study the rotation footprint in complex rotating structures. Applications to non-reciprocal and/or rotation-sensitive metamaterials and optical gyroscopes will be discussed. This research may pave the way for the new paradigm of *rotation-sensitive metamaterials*.

[1] Z. Yu and S. Fan, "Complete Optical Isolation Created by Indirect Inter-band Photonic Transitions," *Nat.Photon*. **3**, 91 (2009)
[2] H. Lira, Z. Yu, S. Fan, and M. Lipson, "Electrically Driven Nonreciprocity Induced by Inter-band Photonic Transition on a Silicon Chip," Phys. Rev. Lett. 109, 033901 (2012).
[3] M. B. Zanjani, A. R. Davoyan, A. M. Mahmoud, N. Engheta, J. R. Lukes, "One-way Phonon Isolation In Acoustic Waveguides," arXiv:1312.0082 (2013)
[4] D. L. Sounas, C. Caloz, and A. Al_u, "Giant Non-Reciprocity at the Subwavelength Scale Using Angular Momentum-Biased Metamaterials," *Nat. Communications*, **4**, 2407 , (2013)
[5] D. Sounas, and A. Alu, "Angular-Momentum-Biased Nanorings to Realize Magnetic-Free Integrated Optical Isolation," *ACS Photonics*, **1**, 198-204, (2014)
[6] J. Van Bladel, *Electromagnetic Fields*, IEEE Press, 2nd Ed., 2007.
[7] T. Shiozawa, "Phenomenological and Electron-Theoretical Study of the Electrodynamics of Rotating Systems," *Proc. IEEE* 61(12), pp. 1694-1702, Dec. 1973.
[8] Ben Z. Steinberg, A. Shamir, and A. Boag, "Two-dimensional Greens function theory for the electrodynamics of rotating medium," *Phys. Rev. E*, 74, pp. 016608 1-9, 2006.